Please get out:

* The Ch. 7 notes from Friday

* Your HW worksheet

On the blank piece of paper, please write

1) Your name at the top, and Ch. 7 & 8

2) Title the paper: **PROBABILITY RULES CHEAT SHEET**

Example:

John Smith Ch. 7 & 8

PROBABILITY RULES CHEAT SHEET

Introductory Vocab (use the book, chapter 7, to get the definitions)

Random (trials)- * Individual outcomes are uncertain

* We see a patter in many trials (in the long run)

Ex: flip coin, simulation Tony Gwyn

Probability- * The chance (likelihood) of something happening

---

Experimental Probability- the probability that ACTUALLY happened in the experiment.

Ex: If I toss a coin 30 times, and get 12 heads, what's the **experimental** prob. of getting heads?

$P(H) = 12/30$

Theoretical Probability- The probability that SHOULD have happened in the experiment.

Ex: Using the same coin tossing situation above, what's the **theoretical** prob. of getting heads?

$P(H) = 0.50 = 50\% = 1/2$

Probability Models- A list of all the possible outcomes and the probability of each outcome.

X	0	1	2	3
P(X)	½	⅓	⅛	. . .

Sample Space- All the possible outcomes (List)

Example: What's the sample space for the Tony Gwyn experiment? Hit or Not hit (miss)

How about for the spinner experiment? Red, Blue, Yellow, Green

What about the rolling 2 dice experiment? 2, 3, 4, 5, ….. , 9, 10, 11, 12

Event- An outcome, or a set of outcomes

Ex: When rolling one dice, outcomes could be: Evens, a 5, etc.

**READ PAGE 318 – 319: "The Myths"

Probability Notation:

- A, B, C, etc. = events that can happen
- P(A) = the probability of event A happening
- S = the sample space

P(H)

P(Jack)

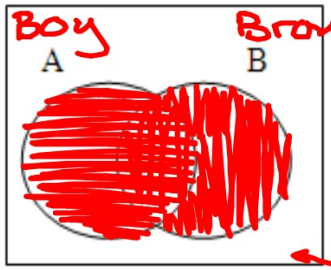
P(

CHAPTER 7: PROBABILITY

General Set Theory

Union:

- Meaning: addition. OR
- Symbol: ∪ A ∪ B
- Example 1:



Sample Space

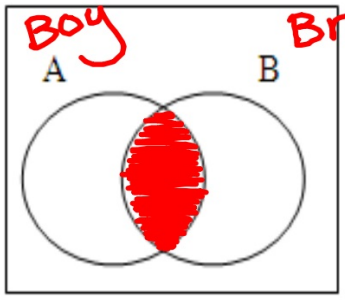
- Example 2: Set A = {2, 4, 6, 8, 10, 12}
  Set B = {1, 2, 3, 4, 5, 6, 7}

  A ∪ B = A or B = { }

{1, 2, 3, 4, 5, 6, 7, 8, 10, 12}

Intersection:

- Meaning: Overlap, AND
- Symbol: ∩ $A \cap B$
- Example 1:

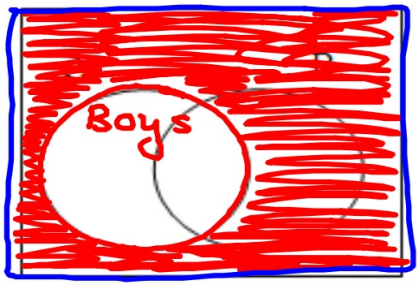


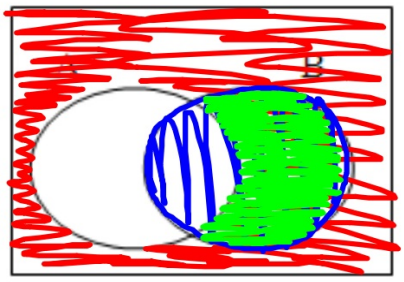
- Example 2: Set A = {2, 4, 6, 8, 10, 12}
  Set B = {1, 2, 3, 4, 5, 6, 7}

  $A \cap B$ = A and B = { 2, 4, 6 }

Complement:

- Meaning: "NOT" An event not happening Brown^c
- Symbol: A^c or A' Boy^c = not a boy
- Example 1: Shade A^c Shade $A^c \cap B$ overlap





- Example 2: Set A = {2, 4, 6, 8, 10, 12}
  S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15} =sample space

A^c = { }

{1, 3, 5, 7, 9, 11, 13, 14, 15}

TRY THESE:

Sample Space = {1, 2, 3, 4, ... 22, 23, 24, 25}

A = {1, 3, 6, 7, 8, 10, 11, 13, 14, 15}

B = {3, 5, 7, 9, 11, 13, 15, 17, 19, 21}

C = {2, 4, 6, 8, 10, 12, 14, 16, 20, 22, 24}

1) What is A ∩ B?

{3, 7, 11, 13, 15}

2) What is A U B?

{1, 3, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 17, 19, 21}.

3) What is B^c?

{1, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 23, 24, 25}

4) What is C ∩ B^c?

{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24}

9) What is P(A ∩ B)?

0

5/25 = 1/5

10) What is P(A U B)?

15/25

11) What is P(B^c)?

15/25

12) What is P(C ∩ B^c)?

12/25

5) What is $A \cap B \cap C$?

empty set

$\{ \emptyset \}$

6) What is $A \cup B \cup C$?

$\{1,2,3,4,5,6,7,8,9,10,11,12,13\ldots,17,19\ldots22,24\}$

all except 18 and 23 and 25

7) What is $A \cap C$?

$\{6,8,10,14\}$

8) What is C^c?

$\{1,3,5,7,9,11,13,15,17,19,21,23,25\}$

13) What is $P(A \cap B \cap C)$?

$0/25 = 0\%$

14) What is $P(A \cup B \cup C)$?

$22/25$

15) What is $P(A)$?

$10/25$

16) What is $P(B)$?

$10/25$

Try these.... **SET THEORY**

S = sample space = {1, 3, 4, 5, 6, 7, 8, 11, 13, 15, 16, 17, 19, 21, 22, 23, 26, 28, 29, 30}
E = {1, 3, 5, 7, 15, 17, 23}
H = {4, 6, 8, 16, 22, 26, 28, 30}
D = {3, 4, 5, 6, 7, 8, 11, 13, 15}
C = {15, 16, 17, 19, 21, 23, 26}
A = {1, 5, 15, 16, 22, 26, 28, 30}
B = {7, 13, 15, 16, 17, 22, 28, 30}

HW

Assuming all data in S are equally likely, find each of the following:

a. P(H) =

b. P(E) =

c. E ∩ H = {

d. P(E ∩ H) =

e. $D \cup B = \{$

f. $P(D \cup B) =$

HW

g. $P(D) =$

h. $P(A) =$

i. $P(A^c) =$

j. $P(C^c) =$

k. $P(B) =$

l. $P(E \cap B) =$

m. $P(D^c) =$

n. $P(A \cap B) =$

o. $P(E \cap C) =$

p. $P(A \cup B) =$

q. $P(E \cup H) =$

r. $P(C \cap A) =$

Probability Rules

- Let A and B be events
- Let S = sample space
- Let A^c = the complement of event A

- List the 3 probability rules:

(1) $0 \leq P(A) \leq 100\%$

(2) $P(S) = 100\%$

(3) $P(A^c) = 100\% - P(A)$

Write these 3 rules on your probability rules cheat sheet!!

Example 1: If the probability of hitting a homerun is 30%, what's the probability of not hitting a homerun?

* $P(H) =$ 30% = 0.30

$P(H^c) =$ 1 − 0.30 = 0.70 or 70%

Example 2: If there are only 8 different blood types, fill in the chart below:

Prob. Model 1 −

Type	A+	A-	B+	B-	AB+	AB-	O+	O-
Probability	0.16	0.14	0.19	0.17	?	0.07	0.1	0.11

0.06

Example 3: Las Vegas Zeke, when asked to predict the ACC basketball Champion, follows the modern practice of giving probabilistic predictions. He says, "UNC's probability of winning is twice Duke's. NC State and UVA each have probability 0.1 of winning, but Duke's probability is three times that. Nobody else has a chance." Has Zeke given a legitimate assignment of probabilities to all the teams in the conference? Why or why not?

P(UNC) = 0.6 P(NCState) = 0.1 1.1 = P(S)

P(Duke) = 0.3 P(UVA) = 0.1

Going back to the examples from before...

Ex #1: There are only 8 different blood types, given in the chart below:

Type	A+	A-	B+	B-	AB+	AB-	O+	O-
Probability	0.16	0.14	0.19	0.17	0.06	0.07	0.1	0.11

What is the probability of being either Type A+ or B-?

$P(A+ \cup B-) = 0.16 + 0.17 = 0.33$

What is the probability of being either Type O- or O+?

$P(O- \cup O+) = P(O) = 0.1 + 0.11 = 0.21$

What is the probability of being either Type AB+ or A+?

$P(AB+ \cup A+) = 0.06 + 0.16 = 0.22$

So... to find the probability of event A OR event B we....

$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$

$P(K) =$

$P(L) =$

Ex #2:

We are picking one card out of a standard 52-card deck (no jokers).

What is the probability of picking a diamond? $P(D) = \frac{13}{52}$

What is the probability of picking a 3? $P(3) = \frac{4}{52}$

What is the probability of picking a diamond or a 3?

$P(D \cup 3) = P(D) + P(3) = \frac{17}{52} - \frac{1}{52}$

$= \frac{16}{52}$

What is the probability of picking a black card?

$P(B) = \frac{26}{52}$

What is the probability of picking a Jack?

$P(J) = \frac{4}{52}$

What is the probability of picking a black card or a Jack?

$P(B \cup J) = \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{28}{52}$

So... to find the probability of event A OR event B we....

$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$

But why didn't we do this in EX #1? What was different about EX #1?

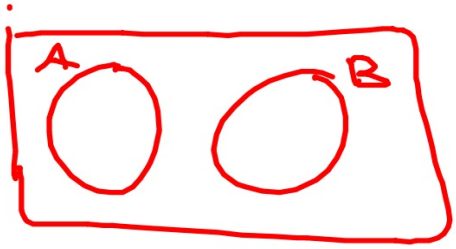
no overlap

This is called... and it means that...

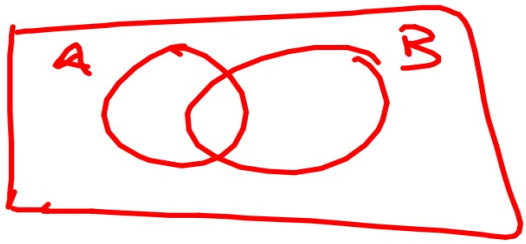
disjoint = no intersection

Visuals: Venn Diagrams

DISJOINT



NOT DISJOINT:



RULE for Unions (OR):

Write these rules down on your probability rules cheat sheet:

Unions:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = 0$

SPECIAL CASE: **Disjoint (A and B do not overlap)

no intersection

$P(A \cup B) = P(A) + P(B)$

Try this:

1) P(K) = 0.34 and P(B) = 0.22 and P(K ∩ B) = 0.13.

   a. Are K and B disjoint? Why or why not?

No. $P(K \cap B) \neq 0$.

   b. What is $P(K \cup B)$?

$P(K \cup B) = P(K) + P(B) - P(K \cap B)$

$= 0.34 + 0.22 - 0.13 = 0.43$

2) P(H) = 0.57 and P(J) = 0.32 and P(J ∩ H) = 0.20.

   a. Are H and J disjoint? Why or why not?

No. $P(H \cap J) \neq 0$.

   b. What is $P(H \cup J)$?

$P(H) + P(J) - P(H \cap J)$

$0.57 + 0.32 - 0.20 = 0.69$

NEW EXAMPLES!

EX #3: The probability of hitting a homerun is 30%. What is the probability of hitting 2 HRs in a row?

$P(H) = 0.30$ $P(H \cap H) = (0.3)(0.3) = 0.09$

How about 3 HRs in a row?

$P(H \cap H \cap H) = 0.3 \cdot 0.3 \cdot 0.3 = 0.027$

How about 4 HRs in a row?

How about 2 HRs then a non-homerun?

$P(H \cap H \cap H^c) = (0.3)(0.3)(0.7) = 0.063$

$(\frac{1}{2})(\frac{1}{2})(\frac{1}{2})$

EX #4: Picking poker chips from a bag ***with*** replacement:

I have a bag that has 10 poker chips in it. 3 are red, 2 are green, and 5 are blue.

$P(R) = 3/10$ $P(G) = 2/10$ $P(B) = 5/10$

What is the probability of picking a red chip and a blue chip?

$$P(R \cap B) = (3/10)(5/10) = 0.15$$

What is the probability of picking a green chip and a red chip?

$$P(G \cap R) = (2/10)(3/10) = 0.06$$

What's the probability of picking a blue chip and a blue chip?

$$P(B \cap B) = (5/10)(5/10) = 0.25$$

Page 8

So... to find the probability of event A <u>AND</u> event B we....

$$P(A \text{ and } B) = P(A \cap B) = P(A) \cdot P(B)$$

PLEASE GET OUT YOUR HW (page 4 in notes)

HW ANSWERS:

(A) 8/20

(C) {∅}

(E) {3, 4, 5, 6, 7, 8, 11, 13, 15, 16, 17, 22, 28, 30}

(G) 9/20

(I) 12/20

(K) 8/20

(M) 11/20

(O) 3/20

(Q) 15/20

∅

P(D U B)

EX #5: Picking poker chips from a bag ***without*** replacement

I have a bag that has 10 poker chips in it. 3 are red, 2 are green, and 5 are blue.

$P(R) = 3/10 \quad P(G) = 2/10 \quad P(B) = 5/10$

What is the probability of picking a red and then a blue?

$$P(R \cap B) = \left(\frac{3}{10}\right)\left(\frac{5}{9}\right) = P(R) \cdot P(B \mid R)$$

($P(B \mid R)$: given that)

What is the probability of picking a blue and then a blue?

$$P(B \cap B) = \left(\frac{5}{10}\right)\left(\frac{4}{9}\right) = P(B) \cdot P(B \mid B)$$

($P(B \mid B)$: given that)

What is the probability of picking a green and then a red?

$$P(G \cap R) = \left(\frac{2}{10}\right)\left(\frac{3}{9}\right) = P(G) \cdot P(R \mid G)$$

~~$P(B \mid BAB)$~~

VOCAB: Conditional Probabilities:

$P(B|A) \neq P(B)$

2nd event | 1st event

So... to find the probability of event A AND event B we....

$P(A \text{ and } B) = P(A \cap B) = P(A) \cdot P(B|A)$

- But why didn't we do this in EX #3 and 4? What was different about EX #5?

with / without replacement

This is called... and it means that...

independent = 1st event doesn't affect 2nd event (replacement)

RULE for Intersections (AND):

Ex: die spinner coin

Write this new rule down on your probability rules cheat sheet:

INTERSECTIONS:

$P(A \cap B) = P(A) * P(B|A)$ or $P(B|A) = \frac{P(A \cap B)}{P(A)}$

↑ given that

SPECIAL CASE: **INDEPENDENT (A happening doesn't affect B happening)

$P(A \cap B) = P(A) * P(B)$

Try these:

1) $P(Y) = 0.35$ and $P(K) = 0.41$ and $P(Y \cap K) = 0.13$.

   a. Are Y and K independent? Why or why not?

No. $P(Y \cap K) \neq P(Y) \cdot P(K)$

$P(Y|K) = P(Y)$

$P(Y \cap K) = P(Y) \cdot P(K)$

$0.13 = (0.35)(0.41)$

$0.13 \neq 0.1435$

   b. What is $P(Y|K)$?

$= \frac{P(Y \cap K)}{P(K)} = 0.317$

   c. What is $P(K|Y)$?

$= \frac{P(K \cap Y)}{P(Y)} = 0.371$

$P(B|A) \neq P(A|B)$

$P(B|A) = \frac{P(A \cap B)}{P(A)}$

2) $P(H) = 0.55$ and $P(D) = 0.37$ and $P(D|H) = 0.20$.

   a. Are H and D independent? Why or why not?

$P(D|H) \neq P(D)$ No.

   b. What is $P(D \cap H)$?

$0.11 = P(D|H) \cdot P(H)$

   c. What is $P(H|D)$?

$0.2973 = \frac{P(H \cap D)}{P(D)} = \frac{0.11}{0.37}$

Try the probability rules worksheet #1

@

① $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.26 + 0.41 - 0.1$

$= 0.57$

ⓑ $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.1}{0.26} = 0.385$

ⓒ disjoint = no overlap

No! $P(A \cap B) \neq 0$

② $P(A \cap B) = P(A) \cdot P(B)$ * indep. *

$= (0.42)(0.33)$

$= 0.1386$

③ @ $P(A \overset{\text{and}}{\cap} B) = P(B|A) \cdot P(A)$

$= (0.2)(0.6) = 0.12$

ⓑ $P(A \overset{\text{or}}{\cup} B) = P(A) + P(B) - P(A \cap B)$

$= (0.6) + (0.34) - (0.12)$

$= 0.82$

a) $\{6, 12\}$

b) $2/20 = 1/10$

c) $\{0, 2, 3, 5, 6, 9, 11, 12, 13, 15, 17, 19\}$

d) $3/20$

e) $10/20$

f) $2/20$

g) $10/20$

h) $\{1, 2, 3, 4 \ldots$

all #'s except

0, 14, 16, 18

Wksht #2

① a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.83$

b) $P(B|A) = \frac{P(B \cap A)}{P(A)} = 0.49$

c) NO. $P(A \cap B) \neq 0$.

② $P(D \cup R) = P(D) + P(R)$ *disjoint*

$= 0.45$

③ @ $P(A \cap B) = P(A) \cdot P(B|A)$

$= 0.09$

ⓑ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.70$

(4) (i) {20}

(j) 1/21

(k)

(l) 3/21

(m) 10/21

(n) 0% = 0/21

(o) 11/21

HW: #1–6

Indep:

$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(B|A) = P(B)$$

disjoint: $P(A \cap B) = 0$